Blackline Masters Table of Contents

$100x - 10x + 270 = 5{,}670$	1. 10% off the original price is $54.
$x - 0.1x = 54$	2. 10% off means paying 90% of the original price. 90% of the original price plus sales tax is $56.70.
$0.9x + 2.70 = 56.70$	3. The price is expressed in cents instead of dollars.

M1.U2.L7 Activity 1: Trading Moves[1] equation cards

1. $-2(x+3) = ½(4x-6)$	2. $3.5x - 3 = 2(x-5)$
3. $0.25x + 1.5 = 2(½x - 3)$	4. $3(2x+1) = -2(1-3x)$

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/teachers/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017-2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Station D: Weekend Earnings

Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay $12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend $7.15 on bus fare.

1. Write three or four mathematical questions that could be solved using this information.

Name: Period: Date:

Station F: Are You Ready For More?

1. Find a pizza place near you and ask about the diameter and cost of at least two sizes of pizza. Compare the cost per square inch of the sizes.

(From Unit 2, Lesson 1)

2. Each figure below is created using stars.

What is the relationship between the figure number and the number of stars in the figure? Represent the relationship in as many ways as possible.

(From Unit 2, Lesson 3)

3. Here is a puzzle:

$$\begin{aligned} m+m &= N \\ N+N &= p \\ m+p &= Q \\ p+Q &= ? \end{aligned}$$

Which expressions could be equal to $p + Q$?

a. $2p + m$

b. $4m + N$

c. $3N$

d. $9m$

(From Unit 2, Lesson 5)

4. Your class is asked to solve this equation.

$$\frac{1}{2}(x+5) = \frac{1}{3}(2x-4)+8$$

Your friend freezes up a bit when they see fractions. Is there an equivalent equation your friend could use that would get the same solution for x but would eliminate the fractions before distributing?

(From Unit 2, Lesson 7)

5. The movie was so boring, Andre walked out of the theater after seeing only a quarter of it. Fifteen minutes later, Mae walked out after seeing a third of it. How long was the movie?[1]

(From Unit 2, Lesson 8)

[1] Adapted from AchievetheCore.org

How many parts of the cup are there?

Which part of the cup matters most over the long run?

If I asked you to tell me how tall a stack of sixty cups would be, what would you do?

If I asked you to go backwards and tell me how many cups are in a 200-centimeter-tall stack, what would you do?

Does it matter if you round to the nearest centimeter?

Teacher Tip:
Print this document front and back.

Hint #2

Hint #1

Hint #4

Hint #3

Hint #5

Teacher Tip:
Print this document front and back.

Preparing for a Modeling Prompt

Ideas for Setting Up an Environment Conducive to Modeling	How to Prepare and Conduct the Modeling Lesson
• Provide plenty of blank whiteboard or chalkboard space for groups to work together comfortably. "Vertical non-permanent surfaces" are most conducive to productive collaborative work. "Vertical" means on a vertical wall, which is better than horizontally on a tabletop, and "non-permanent" means something like a dry erase board, which is better than something like chart paper (Liljedahl 2016). • Ensure that students have easy access to any tools that might be useful for the task. These might include: – supply table containing geometry tools, calculators, scratch paper, graph paper, dry erase markers (ideally a different color for each group member), post-its – electronic devices to access digital tools (like graphing technology, dynamic geometry software, or statistical technology) • Think about how to help students manage the time that is available to work on the task. For example: – Display a countdown timer for intermittent points in the lesson when groups are asked to summarize their progress. – Decide what time to ask groups to transition to writing down their findings in a somewhat organized way (perhaps 15 minutes before the end of the class).	• Decide which version of the prompt students will receive, based on the lift-analysis, timing, and access to data. • Have data ready to share if planning to give it when students ask. • Decide if students will be offered a template for organizing modeling work. • Decide to what extent students are expected to iterate and refine their model. The amount of time available can influence how much time students have to refine their model. If time is short, students may not engage as much in that part of the modeling cycle. WIth more time, it is more reasonable to expect students to iterate and refine their model once or even several times. • Decide how students will report their results. Again, if time is short, this may be a rough visual display on a whiteboard. If more time is available, students might create a more formal report, slideshow, blog post, poster, mockup of an artifact like a letter to a specific audience, smartphone app, menu, or set of policies for a government entity to consider. One way to scaffold this work is to ask students to turn in a certain number of presentation slides: one that states the assumptions made, one that describes the model, and one or more slides with their conclusions or recommendations. • Develop task-specific "look-fors" for each dimension of the provided rubric. What do you anticipate and hope to see in student work?
Organizing Students Into Teams or Groups	**Ways to Support Students While They Work on a Modeling Prompt**
• Mathematical modeling is not a solitary activity. It works best when students have support from each other and their teacher. • Working with a team can make it possible to complete the work in a finite amount of class time. For example, the team may decide it wants to vary one element of the prompt and compute the output for each variation. What would be many tedious calculations for one person could be only a few calculations for each team member. • The members of good modeling groups bring a diverse set of skills and points of view. Create and share a Multiple Abilities List with students. • Scramble the members of modeling teams often, so that students have opportunities to play different roles.	• Coach students on ways to organize their work. • Provide a template to help students organize their thinking. Over time, some groups may transition away from needing to use a template. • Engage students in the *Three Reads* instructional routine to ensure comprehension of the prompt. • Remind students of the variety of tools that are available to them. • If students get stuck or run out of ideas, help move them forward with a question that prompts them to focus on a specific part of the modeling cycle. For example: – "What quantities are important? Which ones change and which ones stay the same?" – "What information do you know? What information would it be nice to know? How could you get that information? What reasonable assumption could you make?" – "What pictures, diagrams, graphs, or equations might help people understand the relationships between the quantities?" – "How are you describing the situation mathematically? Where does your solution come from?" – "Under what conditions does your model work? When might it not work?" – "How could you make your model better? How could you make your model more useful under more conditions?" – "What parts of your solution might be confusing to someone reading it? How could you make it more clear?"

Advice on Modeling[1]

These are some steps that successful modelers often take, and questions that they ask themselves. You don't necessarily have to do all of these steps, or do them in order. Only do the parts that you think will help you make progress.

<table>
<tr><td></td><td>Understand the Question
Think about what the question means before you start making a strategy to answer it. Are there words you want to look up? Does the scenario make sense? Is there anything you want to get clearer on before you start? Ask your classmates or teacher if you need to.</td></tr>
<tr><td></td><td>Refine the Question
If necessary, rewrite the question you are trying to answer so that it is more specific.</td></tr>
<tr><td></td><td>Estimate a Reasonable Answer
If you don't have enough information to decide what's reasonable, try to come up with an answer that would be too low, and an answer that would be too high.</td></tr>
<tr><td></td><td>Identify Unknowns
What are the meaningful quantities in this situation? Write them down.
What information would be useful to know? In order to get that information, you could: look it up, take a measurement, or make an assumption.</td></tr>
<tr><td></td><td>Gather Information
Write down any of the unknown information that you find. As you work, organize your information in a way that makes sense to you.</td></tr>
<tr><td></td><td>Experiment!
Try different ideas to make progress toward answering your question. If you are stuck, think about:
Helpful ways to organize the information you have or organize your work
Questions you can answer using the information you have
Ways to represent mathematical relationships or sets of data (tables, equations, scatter plots, graphs, statistical plots)
Tools that are available for representing mathematics, both digital and analog</td></tr>
<tr><td></td><td>Check Your Reasoning
Do you have a first answer to your question? Great! See if it's reasonable.
Make sure you can explain what the answer means in terms of the original problem.
Check your precision: Is your answer overly precise (do you really need all those decimal places)? Not precise enough (were you overly aggressive with your rounding)?</td></tr>
<tr><td></td><td>Use and Improve Your Model
Did you make assumptions or measurements? How can you express your model more generally, so that it would work for a range of numbers instead of the specific numbers you used?
What are the limitations of your model? That is, what are some ways it is not realistic? Does it only work for certain inputs but not others? Are there any meaningful inputs affecting the outcome that are not accounted for? If possible, improve your model to take these into account.
What are the implications of your model? That is, what should people or organizations do differently or smarter as a result of what your model shows? What would be effective ways to communicate with them?
What are the areas for further research? That is, what new things are you wondering about that could be investigated, by you or someone else?</td></tr>
</table>

[1] Adapted from IM 9-12 Math Algebra 1 Modeling Prompts https://curriculum.illustrativemathematics.org/HS/teachers/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

Modeling Rubric[1]

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">1. Decide What to Model</td><td>• Assumptions made are clearly identified and justified. Resulting limitations are stated when appropriate.
• Variables of interest are clearly identified and chosen wisely, and appropriate units of measure are used.</td><td>• Assumptions are noted but lacking in justification or difficult to find.
• Variables of interest are noted, but may lack justification, be difficult to find, or not be measured with appropriate units.</td><td>• No assumptions are stated.
• No variables are defined.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Ask questions about the situation to understand it better
• Check the assumptions you're making to see if they're reasonable (Try asking a friend, or imagining that you're a person involved in the scenario. Would those assumptions make sense to you?)
• Double-check the variables you've identified: Are there other quantities in the situation that could vary? Is there something you've identified as a variable that is actually fixed or determined? (Remember that more abstract things like time and speed are also quantities.)</td></tr>
<tr><td rowspan="2">2. Formulate a Mathematical Model</td><td>• An appropriate model is chosen and represented clearly.
• Diagrams, graphs, etc. are clear and appropriately labeled.</td><td>Parts of the model are unclear, incomplete, or contain mistakes.</td><td>No model is presented, or the presentation contains significant errors.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Check your model more carefully to make sure it really fits well
• Consider a wider variety of possible models, to find one that fits the situation better
• Think about the situation more deeply before trying to find a model
• Convince a skeptic: Pretend that you think your model is inadequate, or ask a friend to pretend to be skeptical of it. What would a skeptic find wrong with your model? Try to fix those things, or explain why they're not actually problems.</td></tr>
</table>

[1] Adapted from IM 9-12 Math Algebra 1 Modeling Prompts https://curriculum.illustrativemathematics.org/HS/teachers/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

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<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">3. Use Your Model to Reach a Conclusion</td><td>• Solution is relevant to the original problem.
• Reader can easily understand the reasoning leading to the solution.
• Relevant details are included like units of measure.</td><td>Solution is not well-aligned to the original problem, or aspects of the solution are difficult to understand or incomplete.</td><td>No solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Double-check your calculations: Show them to someone else to see if they agree, or take a break and look at your calculations again later
• Make sure your calculations are justified by your model: Ask yourself how you decided what to calculate, and see if your reasoning matches up with your model
• Think more deeply about what your conclusions mean in the original scenario: Imagine you're a person involved in the scenario, or explain your conclusions to someone else and see if they have questions</td></tr>
<tr><td rowspan="2">4. Refine and Share Your Model</td><td>• The model's implications are clearly stated.
• The limitations of the model and solution are addressed.</td><td>The limitations of the model and solution are addressed but lacking in depth or ignoring key components.</td><td>No interpretation of model and solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Think more creatively about what your conclusions mean: Ask yourself "If I was involved in this situation, what would I understand better because of these conclusions? What would I want to do next?"
• Be skeptical of your model: What don't you like about it, and what can you do to fix those things?
• Explain your model to someone else: Tell them how it works and why it's good. If you're not sure how it works or why it's good, you might need to change it.</td></tr>
</table>

Modeling Prompt #1 and Sample Response

What do people mean by “modeling with mathematics”? Here is a prompt that could be approached by modeling, and a sample response to understand and evaluate.

Modeling Prompt

Two friends, Han and Jada, live 7 miles apart. One Saturday, they decide to meet up somewhere between their houses. They each leave their house at 8 a.m. and travel toward each other. They want to choose a place to meet so that they'll both arrive at the same time. Where could they meet?

Sample Student Response

Let's assume we don't have to worry about delays like stop lights or traffic.

Here are some quantities we are working with:

- The speed of each friend is probably miles per hour.
- The distance they each travel (miles)
- The total distance they go (miles) - fixed (7 miles)

To simplify the situation, let's pretend (assume) that they each travel at a constant speed the whole time.

Here are some speeds for different methods of travel (see data sources below):

- Walking 2.5–4 miles per hour
- Driving 25–30 miles per hour
- Biking 10–14 miles per hour

If we assume they each use the same method of transportation and they go about the same speed, they will meet pretty much halfway between their houses. For example, if they both walk at a speed of 3.5 miles per hour, they'd each walk 3.5 miles in an hour and should pick a place to meet up halfway between their houses because 3.5 + 3.5 = 7.

But if they aren't using the same mode of transportation or going the same speed, then they wouldn't meet in the middle. We think the quantities are still the same: total distance covered, distance each friend travels, and speed each friend travels. Also the amount of time it takes them, which is measured in minutes.

Let's say Jada bikes and Han walks. So maybe Jada bikes at 12mph and Han walks 3mph. That means Jada travels 1 mile every 5 minutes, and Han travels .25 miles every 5 minutes. Jada is four times as fast as Han and so they would meet up closer to Han's house because Jada was going faster.

	5	10	15	20	25	30
Han	.25	.50	.75	1	1.25	1.50
Jada	1	2	3	4	5	6

In this scenario, they'll meet in less than 30 minutes because at 30 minutes, they would have traveled 7.5 miles and that is more than the 7 miles between their homes. So, they would meet up sometime between 25 and 30 minutes after they started.

Data Sources

1. https://en.wikipedia.org/wiki/Walking
2. http://infinitemonkeycorps.net/projects/cityspeed/
3. https://www.livestrong.com/article/486666-is-an-average-of-15-miles-per-hour-on-a-bike-good-for-a-beginner/

Display Your Data 2A

Choose a question that you think is interesting and that you do not know the answer to. You will gather data relevant to this question by either doing an experiment or researching available data, so choose a reasonable question.

1. Write your question.

2. What do you predict you will learn from the data?

3. Next, gather your data. Then choose one of these options for displaying your data and answer the questions about it.

   Option 1: Create a display that shows the distribution of the data. Include both measures of center and measures of variability.

   - Why is the data display you selected the best way to summarize your data?
   - What is surprising about the data?
   - Are there any outliers? If so, tell their story. If not, why do you think not?
   - Describe the shape of the distribution.

   Option 2: Use an infographic to summarize what you found.

   - Why is an infographic the best way to summarize your data?
   - What is surprising about the data?
   - What story does your infographic tell?

4. Now that you have seen some data, is there another, related question you would like to study or some additional data you would like to collect?

Display Your Data 2B

1. What are some questions that you might be able to answer with the given data?

2. Choose one of your questions to try to answer. What do you predict the data will tell you?

3. Look for data that's relevant to your question. You might have to change your question a little if you can't find the data you need to answer the original question. When you have an answer, choose a way of displaying the data that will help other people understand what you learned.

   Option 1: Create a display that shows the distribution of the data. Include both measures of center and measures of variability.

   - Why is the data display you selected the best way to summarize your data?
   - What is surprising about the data?
   - Are there any outliers? If so, tell their story. If not, why do you think not?
   - Describe the shape of the distribution.

   Option 2: Use an infographic to summarize what you found.

   - Why is an infographic the best way to summarize your data?
   - What is surprising about the data?
   - What story does your infographic tell?

4. Now that you have seen some data, is there another, related question you would like to study or some additional data you would like to collect?

Here is a student's first draft explanation for Question 3.

$x =- \frac{1}{2}$ is for $\frac{4x-1}{3} = \ - 1$, so as x gets smaller, $\frac{4x-1}{3}$ is also gets smaller. For it to be greater -1, x has to be greater than $- \frac{1}{2}$.

Step 1: Annotate the first draft by marking any parts of the explanation that *you* think are unclear, incomplete, or incorrect. Share your ideas with a partner.

Step 2: Write a second draft of this explanation that is more clear, more complete, and more correct (if needed) than the first draft.

Step 3: Share your second draft with a partner (or your teacher) by reading it aloud. Make any additional improvements to your draft as you read it, if needed.

Name: Period: Date:

End-of-Unit 2 Student Survey

1. Ending this unit I <u>feel</u> ...... (this question could be answered with pictures, words, etc.)

2. How much did you know about the content of this unit before starting?
   a. A great deal b. A little c. Not much

   Feel free to share more:

3. After finishing the unit did your knowledge in the content:
   a. Increase greatly b. Increase a little c. Stay the Same

   Feel free to share more:

4. What was most frustrating for you while learning during this unit?
   a. Materials Used b. Teacher strategies c. Technology d. Other: ________

   Feel free to share more:

5. What boosted your confidence in math during this unit?
   a. Materials Used b. Teacher strategies c. Technology d. Other: ________

   Feel free to share more:

6. What connections do you think the concepts from this unit make to the world around you?

7. What did your level of engagement and participation during the unit tell you about yourself and the way you see yourself and your abilities in math?

8. How would you like to improve in the next unit?

9. How can your teacher support your goals for improvement in the next unit?

10. I'd like my Math 1 teacher(s) to know that I want them to continue ________

11. Please share anything else you'd like regarding your experiences in this unit and your feelings about the upcoming unit.

Graphs, Tables, Equations, and Situations

Card 1

Chicken wings cost $0.60 each.

Let x represent the number of chicken wings purchased. Let y represent the total cost of the chicken wings, in dollars.

Graphs, Tables, Equations, and Situations

Card 2

Clare always checks out the maximum number of items from the library. She can check out up to 15 items (books or DVDs).

Let x represent the number of books Clare checks out. Let y represent the number of DVDs Clare checks out.

Graphs, Tables, Equations, and Situations

Card 3

Tacos are $2 each and dumplings are $1 each. Han plans to spend $10 on snacks.

Let x represent the number of tacos Han could buy and y represent the number of dumplings Han could buy.

Graphs, Tables, Equations, and Situations

Card 4

A city puts a tax on sweetened beverages. The tax is 1.5 cents per ounce.

Let x represent the number of ounces in the drink. Let y represent the tax on the drink, in cents.

Graphs, Tables, Equations, and Situations

Card 5

Kiran runs for 60 minutes a day.

Let x be his average speed for the day, in miles per hour. Let y be the number of miles he runs in a day.

Graphs, Tables, Equations, and Situations

Card 6

A climbing gym charges $50 a month, but gives a permanent $5 discount for every person you refer to the gym.

Let x be the number of people you've referred to the gym and y be your monthly cost, in dollars.

Graphs, Tables, Equations, and Situations

Card 7

Mai has a snow-shoveling business. She charges a flat rate of $50 for the winter, and then an additional $5 for every snowfall over 6 inches.

Let x be the number of snowfalls over 6 inches, and y be the cost of hiring Mai, in dollars.

Graphs, Tables, Equations, and Situations

Card 8

Priya is using 20 meters of fencing to make a rectangular chicken run. She will use the fencing for all 4 sides of the run.

Let x be the length of the run, in meters, and y be the width of the run, in meters.

Graphs, Tables, Equations, and Situations
Card 9

x	y
20	30
12	18
67.6	101.4

Graphs, Tables, Equations, and Situations
Card 10

x	y
2	6
4	2
5	0

Graphs, Tables, Equations, and Situations
Card 11

x	y
6	3.60
10	6.00
12	7.20

Graphs, Tables, Equations, and Situations
Card 12

x	y
3	3
2.5	2.5
3.2	3.2

Graphs, Tables, Equations, and Situations
Card 13

x	y
5	5
4	6
3	7

Graphs, Tables, Equations, and Situations
Card 14

x	y
0	50
2	60
5	75

Graphs, Tables, Equations, and Situations
Card 15

x	y
0	50
2	40
7	15

Graphs, Tables, Equations, and Situations
Card 16

x	y
0	15
7	8
10	5

Graphs, Tables, Equations, and Situations
Card 17

$2x + 2y = 20$

Graphs, Tables, Equations, and Situations
Card 18

$y = 15 - x$

Graphs, Tables, Equations, and Situations
Card 19

$y = 5x + 50$

Graphs, Tables, Equations, and Situations
Card 20

$y = 0.60x$

Graphs, Tables, Equations, and Situations
Card 21

$2x + 1y = 10$

Graphs, Tables, Equations, and Situations
Card 22

$$y = \frac{60x}{60}$$

Graphs, Tables, Equations, and Situations
Card 23

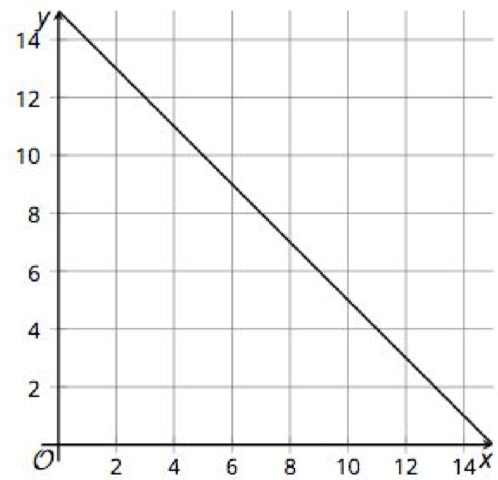
$1.5x = y$

Graphs, Tables, Equations, and Situations
Card 24

$50 - 5x = y$

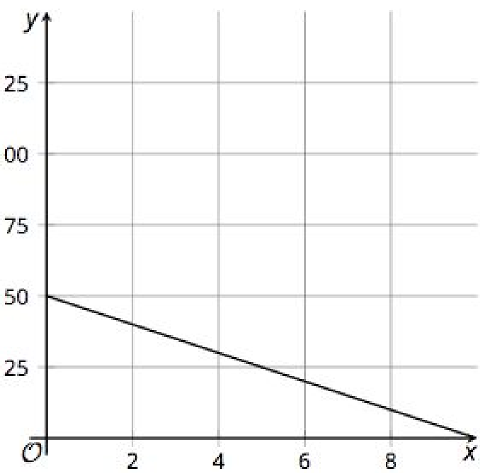
Graphs, Tables, Equations, and Situations

Card 25



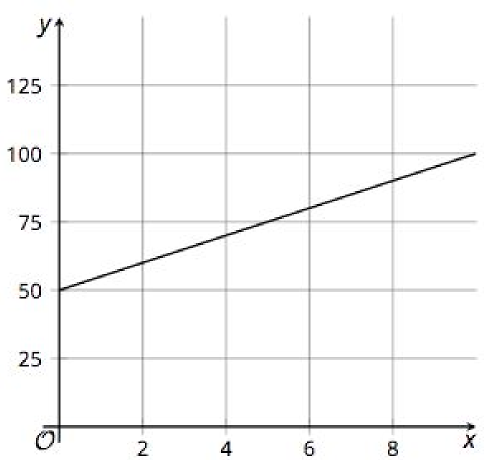
Graphs, Tables, Equations, and Situations

Card 26



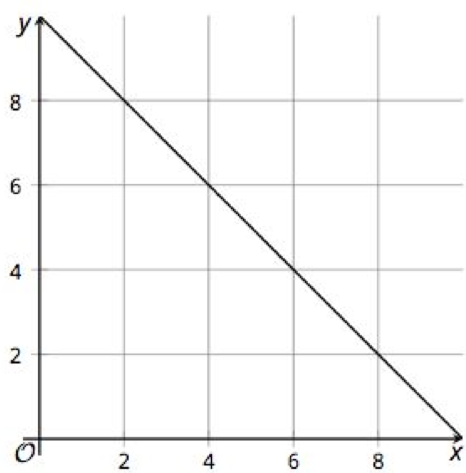
Graphs, Tables, Equations, and Situations

Card 27



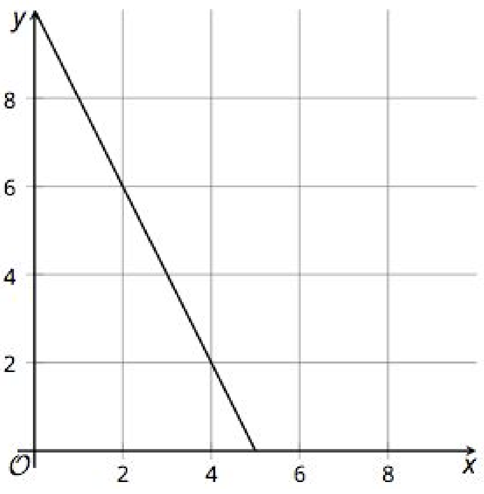
Graphs, Tables, Equations, and Situations

Card 28



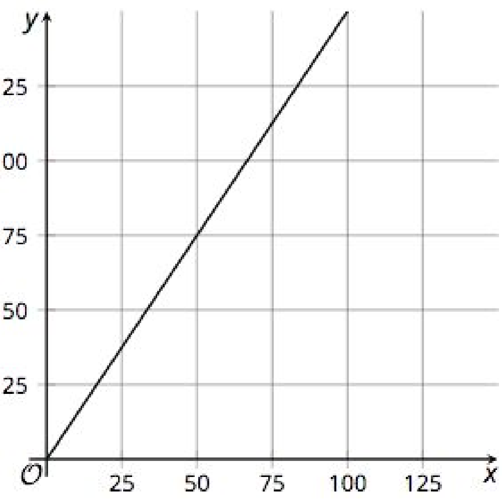
Graphs, Tables, Equations, and Situations

Card 29



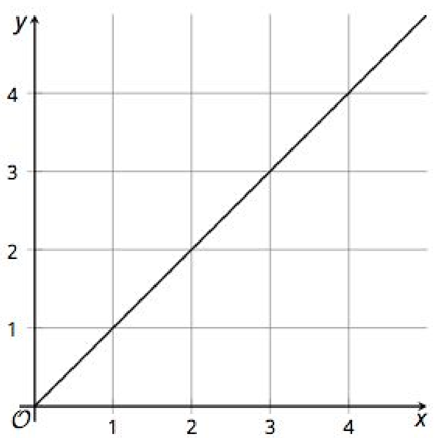
Graphs, Tables, Equations, and Situations

Card 30



Graphs, Tables, Equations, and Situations

Card 31



Graphs, Tables, Equations, and Situations

Card 32